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# RETRIEVAL OF CLOUD LIQUID WATER VERTICAL DISTRIBUTIONS USING COLLOCATED KA-BAND AND W-BAND CLOUD RADARS

Dong Huang[1*], Karen Johnson[1], Yangang Liu[1], and Warren Wiscombe[1,2]

[1]Brookhaven National Laboratory, Upton, NY 11973, U.S.A.
[2]NASA Goddard Space Flight Center (code 913), Greenbelt, MD 20771, U.S.A.
* Corresponding Author: Dong Huang, Environmental Sciences Department, Brookhaven National Laboratory, 75 Rutherford Dr., Upton, NY 11973; Tel (631) 344-5818; Email: dhuang@bnl.gov

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**Brookhaven National Laboratory**
P.O. Box 5000
Upton, NY 11973-5000
www.bnl.gov

## DISCLAIMER

**Abstract**

Previous studies showed that the retrieval of cloud water content using dual-frequency radar attenuation is very sensitive to error in radar reflectivity; either a long radar dwell time or averaging over many range gates would need to reduce random noise in radar data and thus to obtain accurate retrievals but at the cost of poorer temporal and spatial resolution. In this paper we have shown that, by virtue of advanced mathematical inversion techniques like total variation regularization, accurate retrieval of vertically resolved liquid water content at high temporal and spatial resolution is achievable with the co-located Ka-band and W-band cloud radars operated by the Atmospheric Radiation Measurement program. The liquid water path calculated from the dual-frequency retrieval agrees closely with that from a microwave radiometer, with mean difference ranging from 20 to70 $gm^{-2}$ for different cloud cases. Comparison with lidar measurements reveals that the dual-frequency retrieval also reasonably captures the cloud base height of drizzling clouds --- something that is very difficult to determine from radar reflectivity alone.

## 1. Introduction

Low and middle level stratus and stratocumulus are crucial modulators of the Earth's radiation budget because they are optically thick and cover about 46% of the globe on average (Rossow and Schiffer, 1999). Yet, such clouds are poorly represented in numerical models and are considered as one of the largest uncertainties in predictions of climate change. Part of the reason is that existing techniques cannot provide accurate measurements of clouds at the temporal and spatial resolution required for the study of radiation and cloud physical processes.

The potential of millimeter wavelength radar to measure clouds has been recognized for many years (Hobbs et al., 1985; Lhermitte, 1987; Frisch et al., 1995; Kollias, et al., 2005; Matrosov, 2005). In the Rayleigh approximation radar reflectivity is proportional to the sixth power of cloud drop size distribution, but the sixth moment is usually not the most useful parameter for cloud microphysical and cloud radiation transfer studies. In order to obtain more useful moments like cloud liquid water content (LWC) from radar reflectivity, certain assumptions have to be made on cloud drop size distribution. Deviations from these assumptions result in inaccurate relationships between LWC and radar reflectivity (Liu et al., 2008). Furthermore, a small concentration of large drizzle drops can dominate the radar reflectivity yet contribute little to cloud LWC and optical depth. Unfortunately, drizzle is found to be almost ubiquitous at marine and continental stratocumulus clouds from both field campaign studies and satellite observations (Fox and Illingworth, 1997; Mace et al., 2007).

The dual-frequency technique that makes use of differential radar attenuation was therefore proposed to overcome limitations inherent in single-frequency radar techniques to retrieve cloud LWC and effective drop size (Eccles and Mueller, 1971; Martner et al., 1993; Vivekanandan et al., 2001; Hogan et al., 2005). A promising property of the dual-frequency approach is that the difference in the reflectivity factors measured at two different frequencies is directly proportional to the path-integrated LWC and no assumptions on the nature of the cloud drop size distribution are needed, provided only that the cloud drops are small enough to scatter within the Rayleigh regime (< 0.5 mm). A further advantage is that the technique doesn't require absolute calibration of the individual radars; therefore only the capability of measuring the difference in radar reflectivity at two frequencies is needed.

Earlier studies showed that, when 10 and 35 GHz frequencies are used, it is necessary to average over many range gates for a relatively long time period to reduce the random error in radar reflectivity and to gain acceptable retrieval accuracy (Martner et al., 1993; Vivekanandan et al., 2001). For example, the two-way differential attenuation of liquid water with a dual-frequency radar at 10 and 35 GHz is measurable only when the reflectivity factors are averaged over tens of range gates, roughly 4 km (Martner et al., 1993). Hogan et al. (2005) suggested that using 35 and 94 GHz frequencies can substantially improve the retrieval sensitivity; under ideal conditions accurate retrieval of LWC is achievable when the precision of radar measurements is reduced to 0.03 dBZ by increasing the dwell time to one minute and by averaging the data over two range gates (150 m).

Theoretically, prolonged radar dwelling can only reduce the random noise in the data (thus improve the precision of radar reflectivity), but bias errors will not necessarily be damped with a longer dwell time. This poses a challenge to the dual-frequency approach since high resolution retrieval of cloud liquid water is very sensitive to both the random and non-random errors in the radar reflectivity. Advanced mathematical techniques such as total variation regularization have been widely used in solving ill-posed problems and in recovering corrupted noisy digital images. This work adopts such mathematical techniques into the dual-frequency approach and examines the utility of these techniques in two case studies using radar data collected by the Department of Energy Atmospheric Radiation Measurement (ARM) program.

## 2. Methodology

The attenuated radar reflectivity factor $Z_f$, often expressed in conventional logarithmic unit dB$Z$, at frequency $f$ and height $h$ can be calculated from the unattenuated reflectivity factor $Z^u$ at the same height and one-way atmospheric attenuation coefficient $\alpha_f$ (dB km$^{-1}$). The formula can be written as (Hogan et al., 2005):

$$Z_f(h) = Z^u(h) - 2\int_0^h \alpha_f(z)dz \,. \quad (1)$$

Note that the unattenuated reflectivity factor $Z^u$ is not a function of radar frequency $f$, provided that the radar scattering is in the Rayleigh regime. Here we assume any difference between the dielectric factor $|K|^2$ at the frequency $f$ and that at an unattenuated frequency is negligible or is already included in the calculation of radar reflectivity factors.

Assuming $f$ = 35 and 95 GHz respectively in Eq. (1), calculating their difference, and performing a simple manipulation leads to,

$$Z_{35}(h) - Z_{95}(h) = 2\int_0^h \left[\alpha_{95}(z) - \alpha_{35}(z)\right] dz \tag{2}$$

The attenuation of radar signal is mainly due to cloud liquid water absorption, water vapor absorption, and oxygen molecular absorption. The radar attenuation coefficient $\alpha_f$ at level $h$ is a linear function of the mean LWC, denoted as $x$, at the same level,

$$\alpha_f(h) = \kappa_f(h) \cdot x(h) + \alpha^{\text{other}}(h), \tag{3}$$

where $\kappa_f$ is the attenuation efficiency coefficient of liquid water (dB km$^{-1}$ (gm$^{-3}$)$^{-1}$), and $\alpha^{\text{other}}$ is the attenuation by other atmospheric components (water vapor and oxygen). In the Rayleigh approximation, the formulae for calculating these attenuation coefficients for non-precipitating clouds are those of Westwater (1972).

A numerical quadrature for Eq. (2) can be obtained by dividing the cloudy domain into $N$ layers that are equally separated by distance $\Delta h$. Let $h_i$, $h_{i+1}$ be the heights of the bottom and top of layer $i$, and $x_i$ be the mean LWC in the layer $i$. Substituting Eq. (3) to Eq. (2), it is easy to show that the vertical distribution of cloud LWC is related to the difference between radar attenuation at 35 and 95 GHz:

$$\begin{cases} Z_{35}(h_1) - Z_{95}(h_1) - \beta_1 = 2\Delta h(\kappa_{95} - \kappa_{35})x_1 \\ Z_{35}(h_2) - Z_{95}(h_2) - \left(\beta_1 + \beta_2\right) = 2\Delta h(\kappa_{95} - \kappa_{35})(x_1 + x_2) \\ Z_{35}(h_3) - Z_{95}(h_3) - \left(\beta_1 + \beta_2 + \beta_3\right) = 2\Delta h(\kappa_{95} - \kappa_{35})(x_1 + x_2 + x_3) , \\ \qquad\qquad\qquad\qquad \vdots \end{cases} \tag{4}$$

where $\beta_i = 2\Delta h\left[\alpha_{95}^{\text{other}}\left(\frac{h_{i-1}+h_i}{2}\right) - \alpha_{35}^{\text{other}}\left(\frac{h_{i-1}+h_i}{2}\right)\right]$ represents the difference in radar reflectivity caused by the absorption from atmospheric absorptive components other than cloud liquid water. Since the focus of this paper is to examine the validity of the dual-frequency radar method for retrieving vertical distribution of cloud liquid water, we assume the attenuation by water vapor and oxygen can be calculated exactly from nearby radiosonde ascents. Here we neglect the dependence of the absorption efficiency coefficient $\kappa_f$ on height $h$.

The system of equations (4) can be solved analytically given that the radar reflectivity factors can be measured at each layer at 35 and 95 GHz frequencies by a dual-frequency radar. However, many studies have shown that the analytical solution is very sensitive to error in radar reflectivity. For example, for a typical mid-latitude stratocumulus a 0.5 dBZ error in each of the 35 and 95 GHz radar reflectivities corresponds to a 2.8 gm$^{-3}$ error in the LWC retrieval (assuming $\Delta h$ = 50 m), which makes the analytical solution almost useless. Here we convert the retrieval problem of dual-frequency radar into the inversion of a matrix equation (Eq. (5)) so that regularization techniques can be used to improve the retrieval of cloud LWC from noisy radar data.

$$\mathbf{A}\mathbf{x} = \mathbf{b} , \tag{5}$$

where $\mathbf{x}^T = (x_1, x_2, \cdots, x_n)$ is the vector of cloud LWC; $\mathbf{b}^T = (b_1, b_2, \cdots, b_n)$ is the vector of radar differential attenuation where $b_i$ equals the left-hand side of Eq.(4); and $\mathbf{A} = (a_{ij})$ is a triangular matrix with its entry:

$$\alpha_{ij} = \begin{cases} 2\Delta h(\kappa_{95} - \kappa_{35}), & \text{if } i \geq j \\ 0, & \text{otherwise} \end{cases} . \tag{6}$$

Eq. (5) is then solved using the total variation (TV) regularization approach, a widely technique in image denoising applications as well as ill-pose inversion problems whose solution is sensitive to noise. Instead of minimizing the rms difference between predictions (**Ax**) and observations (**b**), the regularized solution minimizes the total variation of the retrieval subject to the data constraint:

$$\min_{x}\{\|\mathbf{x}\|_1\} \text{ subject to } \|\mathbf{Ax}-\mathbf{b}\|_2^2 \le \varepsilon \text{ and } \mathbf{x} \ge 0 . \quad (7)$$

The notation $\|\cdots\|_1$ stands for the $L_1$ norm of a vector, and $\varepsilon$ is an error tolerance usually set to the estimated uncertainties in the measurements and in the forward model.

Unlike the $L_2$ norm Tikhonov regularization that usually penalizes more when the values are large and thus it tends to bias toward a smooth solution (Strong and Chan., 2003), the $L_1$ norm TV regularization doesn't penalize discontinuities in the solution, while simultaneously not penalizing smoothness in the solution either; thus under certain conditions it can preserve the exact discontinuous edge in the solution (Acar and Vogel, 1994; Chambolle and Lions, 1997). A numerical implementation of the TV regularization described in Huang et al. (2009) is used here to solve problem (7). This retrieval algorithm is iterative and it adaptively finds the solution that satisfies the data constraint (within the error tolerance $\varepsilon$) when moving toward the direction of the smallest total variation.

## 3. Data and instruments

The main datastreams used in this study are from the vertically-pointing millimeter wavelength cloud radar (MMCR) and W-band ARM cloud radar (WACR), both of which have been operated at ARM's Southern Great Plains (SGP) central facility for years. The SGP central facility is located on 160 acres of cattle pasture and wheat fields southeast of Lamont, Oklahoma, and it is heavily instrumented with a variety of cloud, radiation, and aerosol instruments.

### 3.1 Millimeter Wavelength Cloud Radar

The MMCR is a vertically-pointing radar that operates at a frequency of 35 GHz (8 mm). The radar has a 0.2° beamwidth. The MMCR provides radar reflectivity of the atmosphere with vertical resolutions of 45 or 90 m and height coverage from 0.1 up to 15 km above ground level. The radar also possesses a Doppler capability that allows the measurement of cloud constituent vertical velocities.

The MMCR cycles through several distinct operating modes, each optimized for specific types and locations of clouds and precipitation. The focus of this study is to examine the validity of the dual-frequency radar technique for retrieving cloud LWC profiles, so we use the data from only the boundary layer mode (mode 1). The MMCR switches to this mode every 4 s to sample only the lowest kilometers (up to 4.5 kilometers above ground level), but has better sensitivity there than the other modes. Under the boundary layer operating mode, the dwell and processing time is 2 s and the reflectivity measurements are accurate to within 0.5–1.0 dB. The vertical resolution of the measurements is 45 m.

### 3.2 W-band ARM Cloud Radar

The WACR system is a zenith-pointing Doppler radar at 95 GHz (3.15 mm). The WACR is installed in the same shelter as the 35 GHz MMCR in order to maximize overlap (a few meters separation). The beam width of the WACR is 0.35° and the vertical resolution is about 43 m. The estimated uncertainty of measured reflectivity is about 0.5 dB. The system sensitivity at 2 km is -45 dB with 2 s average. After each 2-s acquisition, the system performs an internal calibration to monitor receiver gain, noise figure and transmitter output power. The WACR does not use pulse coding and operates in only copolarization and cross-polarization modes. The data from the copolarization mode are used in this study; this means every 4 s we get a co-polarization measurement and in between a cross-polarization one.

### 3.3 Lidar cloud base height

The Active Remote Sensing of CLouds (ARSCL) value-added product (VAP) combines data from active remote sensors to produce an objective determination of hydrometeor height distributions and estimates of their radar reflectivities, vertical velocities, and Doppler spectral widths, which are optimized for accuracy (Clothiaux et al., 2000). The ARSCL cloud base height will be used in this study to examine the ability of the dual-frequency radar attenuation technique to identify the low water content drizzling regions below cloud base (these regions usually show high radar reflectivity). The determination of cloud base height in the ARSCL algorithm relies on the commercial Vaisala laser ceilometer and a micropulse lidar located at the SGP facility (Cothiaux et al., 2000).

### 3.4 Microwave Radiometer

The MWR measures the downwelling microwave radiant energy of the sky (usually converted to brightness temperature for convenience) at 23.8 and 31.4 GHz frequencies. Cloud liquid in the atmosphere emits in a continuum that increases with frequency, constituting the primary portion of the signal at the 31.4 GHz channel, whereas water vapor dominates the signal at 23.8 GHz. The water vapor and liquid water signals can, therefore, be separated by observing at these two frequencies. The beam widths are unequal for the two frequencies: 5.5° at 23.8 GHz and 4.6° at 31.4 GHz. The sampling time of the MWR is 20 s and this gives a precision of 0.3 K in the measurements of microwave brightness temperature. The retrieval accuracy of the liquid water path (LWP) under low and intermediate liquid water conditions is about 30 $gm^{-2}$. (Turner et al., 2007)

## 4. Results

We select two very different cloud cases to examine the skill of this dual-frequency radar technique. We use the data from the collocated ARM Ka- and W-band radars at the SGP central facility to retrieve cloud LWC profiles, and use the LWP retrievals from the nearby microwave radiometer and the ARSCL cloud base heights to validate the dual-frequency retrievals.

### 4.1 January 20 2006 case

We first present the retrievals of stratocumulus LWC at the SGP central facility site, on January 20, 2006. The MMCR and WACR are separate radars rather than a true dual-frequency radar, i.e,. they are not ideally synchronized, their beamwidths, gate lengths and sampling rates are different. Data from both instruments must first be interpolated to a common time and space grid. We choose a temporal resolution of 4 s since this is close to the sample rate of each of the radar operating modes of interest in this study. The vertical resolution is set to 46 m. To remove the effects of possible reflectivity drift, we adjust the MMCR data so that they match those of WACR at the first two range gates that show significant radar return. Figures 1a&b depict the 35 and 95 GHz reflectivity fields between 1200 and 2400 UTC overlaid by the ARSCL cloud base height. Note that we use the temperature and pressure fields from the ARM merged sounding Value-Added Product and subtract the gaseous attenuation (water vapor and oxygen) from the radar reflectivities by using the water vapor mixing ratio calculated by assuming 100% relative humidity in clouds. Figures 1a&b show that the cloud layer is characterized by variable cloud top and base, with a significant presence of drizzle down to the surface from 1800 to 2400 UTC.

The LWC field retrieved using the algorithm of Eq. (7) is shown in Figure 1c. It appears that most of the LWC is located in the upper cloud. A general increase in LWC with height is apparent, which is typical of well-mixed stratocumulus. The maximum LWC found in the cloud is 1.0 $gm^{-3}$. Figure 1d compares the LWP calculated from the retrieved LWC field with the LWP obtained by the MWR. The dual-frequency radar LWP shows substantially more variation than the MWR LWP, since the MWR beamwidth is one order wider that the beamwidth of the radars. The agreement is good, with a mean difference of 18 $gm^{-2}$ and an rms difference of 29 $gm^{-2}$. But this agreement, of course, does not guarantee the accuracy of the vertical distribution of cloud LWC. By overlaying the ARSCL cloud base with the LWC retrieval (Figure 1c), we see that the dual-frequency retrieval reasonably identifies the drizzling region of low liquid water, while is impossible to distinguish using the radar reflectivity images alone (Figures 1a&b).

4.2 May 6 2006 case

Now we present observations made by the two radar at the SGP site on May 6, 2006. The data are processed in the same manner as the previous case. Figures 2a&b depict the radar reflectivity fields observed at 35 and 95 GHz between 1200 and 2400 UTC in a convective cloud. Updrafts and downdrafts of intermediate strength were observed from 1200 to 1700 UTC and the cloud showed very complicated structure during this period. High reflectivity factors are seen at all levels, indicating ubiquitous drizzle presence.

Figure 2c shows the retrieved LWC field along with the cloud base height derived from lidar returns by the ARSCL algorithm. The retrieval seems to capture the cloud base height very well in this case, in spite of strong drizzle returns below cloud base from 1200 to 1800 UTC. Figure 2d shows that the time series of LWP from the dual-frequency retrieval agrees faithfully with that of the MWR LWP during the period of 1700 to 2400 UTC. From 1200 to 1700 UTC, the dual-frequency LWP appears to be substantially larger than the MWR LWP, which is indicative of Mie scattering effects from precipitation particles. The mean difference is 70 $gm^{-2}$, which is three times higher than for the January 20 2006 case. For such an optically-thick cloud (LWP > 1000 $gm^{-2}$) the microwave radiometer will approach saturation and thus the microwave retrieval accuracy is also likely to be degraded.

## 5. Conclusions

The dual-frequency radar approach takes advantage of the fact that the difference in radar attenuation at 35 and 95 GHz frequencies is directly proportional to the total amount of cloud LWC in the involved volume. The differential attenuation is about 7.1 dB $km^{-1}$ $(gm^{-3})^{-1}$ under a typical environmental condition and this means the retrieved LWC is accurate only to within 2.8 $gm^{-3}$ assuming 50 m vertical resolution of the retrieval and 0.5 dBZ uncertainty in the radar reflectivity factors. A long radar dwell time and averaging data over many range gates are thus needed in order to improve the precision of radar measurements and to obtain accurate retrievals. However this degrades the temporal and spatial resolution of the retrievals. In this paper we take a different approach, employing an advanced mathematical inversion technique, called total variation regularization. We demonstrate that accurate retrieval of vertically resolved cloud LWC at high temporal and spatial resolution is achievable with ARM's co-located Ka-band and W-band cloud radars.

We select two cases to examine the dual-frequency radar technique: a low level stratocumulus, and a post-precipitation convective cloud. The LWP calculated from the retrieved LWC profiles agree closely with those retrieved with the MWR, with mean difference ranging from 18 to 70 $gm^{-2}$. Despite that the beamwidths of the two radars and the microwave radiometer differ by more than 10x, the agreement between the dual-frequency retrieval and the microwave radiometer retrieval is very good. This agreement, of course, doesn't guarantee the validity of the retrieved LWC profiles. The validity of one aspect of the profiles, cloud base height, is clear however. The dual-frequency retrievals appear to reasonably capture cloud base heights compared with the ARSCL retrievals, though cloud base is difficult to identify for a drizzling cloud with radar reflectivity alone. Further validation of the dual-frequency radar retrieval requires concurrent independent observations of cloud water profile either by in-situ airborne cloud sensors or by a network of surface-based microwave radiometers using the cloud tomography approach (Huang et al., 2008).

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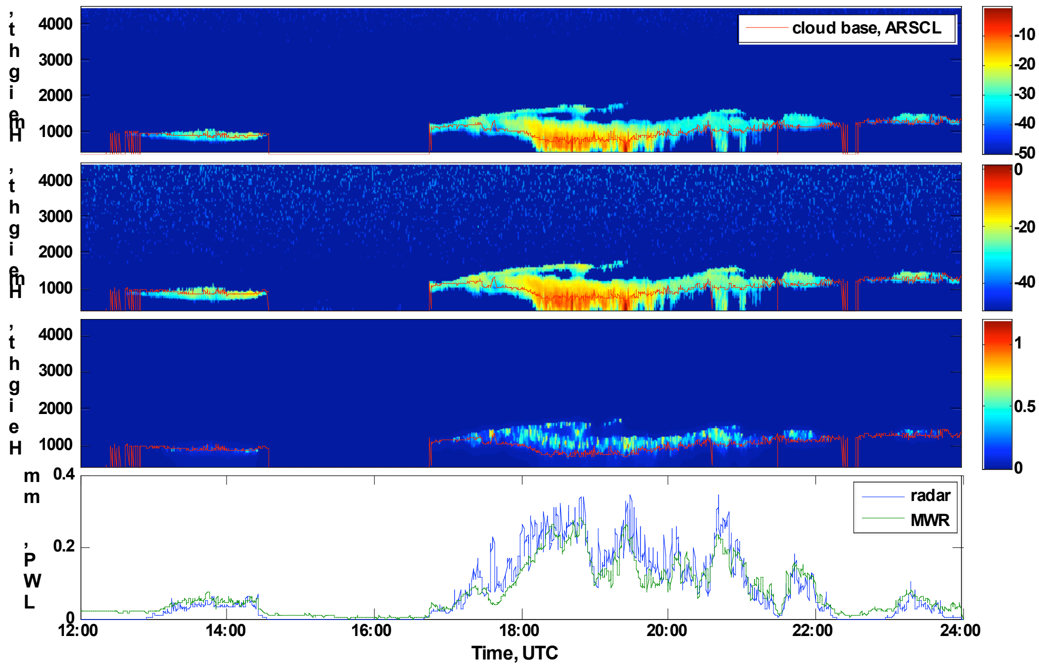


**Figure 1.** Dual-frequency measurements at the Southern Great Plains central facility site on January 20, 2006: (a) radar reflectivity factor at 95 GHz by the WACR with the lidar cloud base shown by red line; (b) radar reflectivity factor at 35 GHz by the MMCR; (c) the retrieved cloud LWC. The data are averaged to 20 seconds to produced the retrieval; (d) the comparison between the dual-frequency retrieved LWP and microwave radiometer LWP.

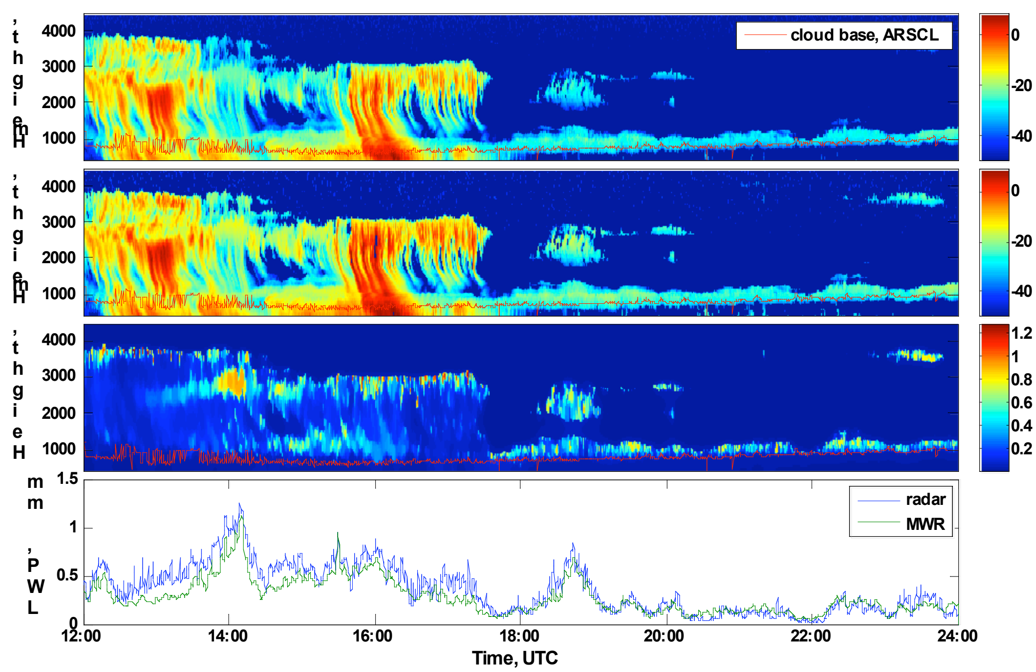


**Figure 2.** Same as Figure 1, but for May 06, 2006.